

# Rotating Dyonic Black Holes in Heterotic String Theory

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We study a class of rotating dyonic black holes in the heterotic string theory in four dimension which have left, right independent electric charges but have same magnitude for the left and right magnetic charges. In both left and right sector the electric and the magnetic vectors are orthogonal to each other. The gyromagnetic (electric) ratios are in general found not to have an upper bound.

## 1. Introduction

In a recent paper [1], we analyzed dyonic black holes in heterotic string theory on a six torus. In particular, we discussed the local as well global structure of those dyonic black holes which are spherically symmetric<sup>1</sup>. It was found that a generic solution of this class depends on four boost parameters which are responsible for left, right electric and magnetic charges. Also it depends on fifty two rotation parameters which set the directions of electric and magnetic charge vectors. When we allow these black holes to have angular momentum, the resulting situation becomes more complicated. Even though, in principle, the local structure of the metric as well as of the gauge fields and scalars can be found from our general discussion [1], we only determined mass, angular momentum and charges looking at the asymptotic behavior of the configuration. This, in turn, allowed us to figure out the cases when the black holes are extremal, BPS saturated and supersymmetric.

In this letter we find and analyse a particular class of rotating dyonic black holes, for which we explicitly give the metric and various other fields. This class of dyons are characterised by independent left, right electric charges but they have same magnitude of the left and right magnetic charges. Among other things, we find they have non-zero gyromagnetic and gyroelectric ratios. However, unlike the pure electrically charged black holes and black holes in pure Einstein-Maxwell theory [3,4], these ratios *do not* have upper bound 2. They can be arbitrarily large. We discuss how to arrive at the solutions and study their properties in the next section.

## 2. Rotating Dyonic Black Holes

It is well known that as long as we consider static configurations of four dimensional heterotic string theory on six torus, the T-duality group and the electric-magnetic duality group of theory combines in a non-trivial way resulting an  $O(8, 24)$  symmetry of the theory [5]. It is also known [3] that the coset  $(O(22, 2) \times O(6, 2))/(O(22) \times O(6) \times SO(2))$  of the original  $O(8, 24)$  acts as a solution generating group. Namely, the action of the coset on a known solution can be used to generate non-trivial field configuration of the theory. In [1] we discussed, in detail, how to parametrize the coset. The explicit procedure for construction of *all* rotating black holes in heterotic string theory on six torus, for fixed asymptotic configuration of various fields representing asymptotically flat space time, was

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<sup>1</sup> The spherically symmetric case has also been analyzed in [2].

also spelled out there, though, as mentioned in the introduction, final configurations were only worked out in the case of static black holes. In what follows, we will explicitly construct a class of rotating dyonic black holes and discuss their properties.

We start with the Kerr metric

$$\begin{aligned}
ds^2 = & -\frac{\rho^2 + a^2 \cos^2 \theta - 2m\rho}{\rho^2 + a^2 \cos^2 \theta} dt^2 + \frac{\rho^2 + a^2 \cos^2 \theta}{\rho^2 + a^2 - 2m\rho} d\rho^2 + (\rho^2 + a^2 \cos^2 \theta) d\theta^2 \\
& + \frac{\sin^2 \theta}{\rho^2 + a^2 \cos^2 \theta} [(\rho^2 + a^2)(\rho^2 + a^2 \cos^2 \theta) + 2m\rho a^2 \sin^2 \theta] d\phi^2 \\
& - \frac{4m\rho a \sin^2 \theta}{\rho^2 + a^2 \cos^2 \theta} dt d\phi, \\
\Phi = & 0, \quad B_{\mu\nu} = 0, \quad A_\mu^{(a)} = 0, \quad M = I_{28},
\end{aligned} \tag{2.1}$$

and apply a properly chosen solution generating matrix belonging to the coset discussed above to get a new rotating black hole geometry. Note that the above configuration is guaranteed to be a solution of the equations of motion of heterotic string theory on six torus since these equations of motion turn out to be identical to Einstein equation in the matter free space. We will not repeat the procedure here but only mention that the matrix which generate rotating dyonic black holes with independent 28 electric and 28 magnetic charges are parametrised, among others, by four boost parameters that we denoted by  $\alpha, \beta, \gamma$  and  $\delta$  and also rotation parameters  $R, T$  and  $u$ .<sup>2</sup> The explicit class of configuration that we generate here are the one which can be obtained by boosting the above Kerr solution where the boost parameters are  $\alpha, \gamma$  and  $\beta(= \delta)$ . We also set the rotation parameters  $R, T$  and  $u$  to zero. It is easy to check that this choice of parameters automatically satisfies the Taub-NUT condition given in [1], and hence guarantees the asymptotically flat metric for the new configuration. Following the procedure of [3,1], after long algebraic manipulations, we get the new metric in the following form:

$$\begin{aligned}
ds^2 = & \tilde{R} \{ -\Delta^{-1} R dt^2 + (\rho^2 + a^2 - 2m\rho)^{-1} d\rho^2 + d\theta^2 + \Delta^{-1} \sin^2 \theta [\Delta \\
& + a^2 \sin^2 \theta \{ \tilde{R} + 2m\rho \cosh \alpha \cosh \gamma - m^2 \sinh^2 \beta (\cosh \alpha - \cosh \gamma)^2 \}] d\phi^2 \\
& - 2\Delta^{-1} m\rho a \sin^2 \theta \cosh \beta (\cosh \alpha + \cosh \gamma) dt d\phi \},
\end{aligned} \tag{2.2}$$

where

$$\begin{aligned}
\Delta = & \tilde{R} (R + 2m\rho \cosh \alpha \cosh \gamma + m^2 (\cosh \alpha - \cosh \gamma)^2) \\
& - m^2 a^2 \cos^2 \theta \cosh^2 \beta (\cosh \alpha - \cosh \gamma)^2 \\
R = & \rho^2 - 2m\rho + a^2 \cos^2 \theta \quad \tilde{R} = \rho^2 + 2m\rho \sinh^2 \beta + a^2 \cos^2 \theta.
\end{aligned} \tag{2.3}$$

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<sup>2</sup> The most general form of the matrix was given in [1]

Similarly we can extract the exact expressions for various fields in the transformed solutions. The dilaton is given by

$$\phi = \frac{1}{2} \log \frac{\tilde{R}^2}{\Delta}. \quad (2.4)$$

The time components of the gauge fields are ( with  $1 \leq n \leq 22$  and  $23 \leq p \leq 28$ )

$$A_t^{(n)} = \frac{R_{22}}{\sqrt{2}} \frac{1}{\Delta} \begin{pmatrix} 0_{20} \\ -ma \cos \theta \sinh \beta (\tilde{R} \cosh \gamma + m \rho \cosh^2 \beta (\cosh \alpha - \cosh \gamma)) \\ m \sinh \alpha \{ m a^2 \cos^2 \theta \cosh^2 \beta (\cosh \alpha - \cosh \gamma) - \tilde{R} (m (\cosh \alpha - \cosh \gamma) + \rho \cosh \gamma) \} \end{pmatrix} \quad (2.5)$$

$$A_t^{(p)} = \frac{R_6}{\sqrt{2}} \frac{1}{\Delta} \begin{pmatrix} 0_4 \\ ma \cos \theta \sinh \beta (\tilde{R} \cosh \alpha - m \rho \cosh^2 \beta (\cosh \alpha - \cosh \gamma)) \\ m \sinh \gamma \{ \tilde{R} (m (\cosh \alpha - \cosh \gamma) - \rho \cosh \alpha) - m a^2 \cos^2 \theta \cosh^2 \beta (\cosh \alpha - \cosh \gamma) \} \end{pmatrix}$$

whereas the spatial components of the gauge fields are as follows

$$A_\phi^{(n)} = \frac{R_{22}}{2\sqrt{2}} \frac{1}{\Delta} \begin{pmatrix} 0_{20} \\ m \cos \theta \sinh 2\beta [\tilde{R} \{ \rho^2 + a^2 + 2m\rho (\cosh \alpha \cosh \gamma - 1) \} + m^2 (\cosh \alpha - \cosh \gamma)^2 \{ \rho^2 + (2m\rho - a^2) \sinh \beta \} + m\rho a^2 \sin^2 \theta (\cosh \alpha \cosh \gamma - \cosh^2 \gamma)] \\ 2m\rho a \sinh \alpha \cosh \beta \sin^2 \theta [\tilde{R} + m (\cosh \alpha \cosh \gamma - \cosh^2 \gamma) \{ \rho + 2m \sinh^2 \beta \}] \end{pmatrix} \quad (2.6)$$

$$A_\phi^{(p)} = \frac{R_6}{2\sqrt{2}} \frac{1}{\Delta} \begin{pmatrix} 0_4 \\ -m \cos \theta \sinh 2\beta [\tilde{R} \{ \rho^2 + a^2 + 2m\rho (\cosh \alpha \cosh \gamma - 1) \} + m^2 (\cosh \alpha - \cosh \gamma)^2 \{ \rho^2 + (2m\rho - a^2) \sinh \beta \} + m\rho a^2 \sin^2 \theta (\cosh \alpha \cosh \gamma - \cosh^2 \alpha)] \\ 2m\rho a \sinh \gamma \cosh \beta \sin^2 \theta [\tilde{R} + m (\cosh \alpha \cosh \gamma - \cosh^2 \alpha) \{ \rho + 2m \sinh^2 \beta \}] \end{pmatrix}$$

where  $R_{22}$  and  $R_6$  are general  $22 \times 22$  and  $6 \times 6$  rotation matrices. The scalar fields are given by

$$M_{ab} = I_{28} + \begin{pmatrix} R_{22} \mathcal{P} R_{22}^T & R_{22} \mathcal{Q} R_6^T \\ (R_{22} \mathcal{Q} R_6^T)^T & R_6 \mathcal{R} R_6^T \end{pmatrix} \quad (2.7)$$

$\mathcal{P}$ ,  $\mathcal{Q}$  and  $\mathcal{R}$  can be written as follows

$$\mathcal{P} = \begin{pmatrix} 0_{20 \times 20} & 0 & 0 \\ 0 & p_1 & p_2 \\ 0 & p_2 & p_3 \end{pmatrix} \quad \mathcal{Q} = \begin{pmatrix} 0_{4 \times 20} & 0 & 0 \\ 0 & q_1 & q_2 \\ 0 & q_3 & q_4 \end{pmatrix} \quad \mathcal{R} = \begin{pmatrix} 0_{4 \times 4} & 0 & 0 \\ 0 & r_1 & r_2 \\ 0 & r_2 & r_3 \end{pmatrix} \quad (2.8)$$

In the above we have defined the following notations:

$$\begin{aligned} p_1 &= \frac{2m^2 \sinh^2 \beta}{\Delta} [\sinh^2 \beta \{\rho^2 - 2m\rho(1 - \cosh \alpha \cosh \gamma) \\ &\quad m^2 (\cosh \alpha - \cosh \gamma)^2\} + \sinh^2 \gamma a^2 \cos^2 \theta] \\ p_2 &= \frac{-2m^2 a \cos \theta \sinh \alpha \sinh \beta}{\Delta} [\rho (\cosh^2 \beta - \cosh^2 \gamma) + m \sinh^2 \beta (\cosh \alpha - \cosh \gamma) \cosh \gamma] \\ p_3 &= \frac{2m^2 \sinh^2 \alpha}{\Delta} [\rho^2 \sinh^2 \gamma + \sinh^2 \beta (2m\rho \sinh^2 \gamma + a^2 \cos^2 \theta)] \\ q_1 &= \frac{-2m \sinh^2 \beta}{\Delta} [\sinh^2 \beta \{m\rho^2 - 2m^2 \rho(1 - \cosh \alpha \cosh \gamma) \\ &\quad + m^3 (\cosh \alpha - \cosh \gamma)^2\} + \rho^3 - ma^2 \cos^2 \theta (1 + \cosh \alpha \cosh \gamma) \\ &\quad + \rho a^2 \cos^2 \theta + m^2 \rho (\cosh \alpha - \cosh \gamma)^2 - 2m\rho^2 (1 - \cosh \alpha \cosh \gamma)] \\ q_2 &= \frac{2ma \cos \theta \sinh \beta \sinh \gamma}{\Delta} [\rho^2 - m\rho(1 - \sinh^2 \beta - \cosh \alpha \cosh \gamma) \\ &\quad + a^2 \cos^2 \theta - m^2 \cosh \alpha (\cosh \alpha - \cosh \gamma) \sinh^2 \beta] \\ q_3 &= \frac{2ma \cos \theta \sinh \alpha \sinh \beta}{\Delta} [\rho^2 + m\rho (\sinh^2 \beta + \cosh \alpha \cosh \gamma) \\ &\quad a^2 \cos^2 \theta + m^2 \sinh^2 \beta \cosh \gamma (\cosh \alpha - \cosh \gamma)] \\ q_4 &= \frac{-2m \sinh \alpha \sinh \gamma}{\Delta} [\rho^3 + m\rho^2 (\cosh \alpha \cosh \gamma - 1 + 2\sinh^2 \beta) \\ &\quad + 2m^2 \rho \sinh^2 \beta (\cosh \alpha \cosh \gamma - 1) + a^2 \cos^2 \theta (\rho + m \sinh^2 \beta)] \\ r_1 &= \frac{2m^2 \sinh^2 \beta}{\Delta} [\sinh^2 \beta \{\rho^2 - 2m\rho(1 - \cosh \alpha \cosh \gamma) + m^2 (\cosh \alpha - \cosh \gamma)^2\} \\ &\quad + \sinh^2 \alpha a^2 \cos^2 \theta] \\ r_2 &= \frac{-2m^2 a \cos \theta \sinh \beta \sinh \gamma}{\Delta} [\rho (\cosh^2 \alpha - \cosh^2 \beta) \\ &\quad + m \sinh^2 \beta \cosh \alpha (\cosh \alpha - \cosh \gamma)] \\ r_3 &= \frac{2m^2 \sinh^2 \alpha}{\Delta} [\rho^2 \sinh^2 \alpha + (2m\rho \sinh^2 \alpha + a^2 \cos^2 \theta) \sinh^2 \beta] \end{aligned} \quad (2.9)$$

The antisymmetric tensor field is given by

$$\begin{aligned} B_{t\phi} &= \frac{\cosh \beta (\cosh \alpha - \cosh \gamma)}{\Delta} [m\rho a \sin^2 \theta \{\tilde{R} + m(\cosh \alpha \cosh \gamma - 1)(\rho + 2m \sinh^2 \beta)\} \\ &\quad - \sinh^2 \beta m^2 a \cos^2 \theta (\rho^2 + a^2 - 2m\rho)]. \end{aligned} \quad (2.11)$$

To get various charges associated with the black hole, we rewrite the metric in the Einstein frame

$$\begin{aligned}
ds_E^2 = & e^{-\Phi} ds^2 \\
= & \sqrt{\Delta} \{ -\Delta^{-1} (\rho^2 - 2m\rho + a^2 \cos^2 \theta) dt^2 \\
& + (\rho^2 + a^2 - 2m\rho)^{-1} d\rho^2 + d\theta^2 \\
& + \Delta^{-1} \sin^2 \theta [\Delta + a^2 \sin^2 \theta \{ \rho^2 + a^2 \cos^2 \theta + 2m\rho \cosh \alpha \cosh \gamma \\
& + m(\cosh^2 \beta - 1)(2\rho - m(\cosh \alpha - \cosh \gamma)^2) \}] d\phi^2 \\
& - 2\Delta^{-1} m\rho a \sin^2 \theta \cosh \beta (\cosh \alpha + \cosh \gamma) dt d\phi \}
\end{aligned} \tag{2.12}$$

This metric clearly corresponds to a class of rotating black holes characterized by different values of boost parameters  $\alpha, \beta$  and  $\gamma$ . The mass  $M$ , angular momentum  $J$ , electric charges  $Q_{elc}^{L,R}$ , magnetic charges  $Q_{mag}^{L,R}$ , electric and magnetic dipole moments  $\mu_{elc}^{L,R}$  and  $\mu_{mag}^{L,R}$  are found to be as follows

$$M = \frac{1}{2} m (\cosh^2 \beta + \cosh \alpha \cosh \gamma) \tag{2.13}$$

$$J = \frac{1}{2} m a \cosh \beta (\cosh \alpha + \cosh \gamma) \tag{2.14}$$

$$Q_{elc}^L = \frac{m}{\sqrt{2}} \sinh \alpha \cosh \gamma R_{22} \begin{pmatrix} 0_{20} \\ 0 \\ 1 \end{pmatrix}, \quad Q_{elc}^R = \frac{m}{\sqrt{2}} \sinh \gamma \cosh \alpha R_6 \begin{pmatrix} 0_4 \\ 0 \\ 1 \end{pmatrix}. \tag{2.15}$$

$$Q_{mag}^L = \frac{m}{2\sqrt{2}} \sinh 2\beta R_{22} \begin{pmatrix} 0_{20} \\ 1 \\ 0 \end{pmatrix}, \quad Q_{mag}^R = -\frac{m}{2\sqrt{2}} \sinh 2\beta R_6 \begin{pmatrix} 0_4 \\ 1 \\ 0 \end{pmatrix}. \tag{2.16}$$

$$\mu_{elc}^L = \frac{ma}{\sqrt{2}} \sinh \beta \cosh \gamma R_{22} \begin{pmatrix} 0_{20} \\ 1 \\ 0 \end{pmatrix}, \quad \mu_{elc}^R = -\frac{ma}{\sqrt{2}} \sinh \beta \cosh \alpha R_6 \begin{pmatrix} 0_4 \\ 1 \\ 0 \end{pmatrix}. \tag{2.17}$$

$$\mu_{mag}^L = \frac{ma}{\sqrt{2}} \sinh \alpha \cosh \beta R_{22} \begin{pmatrix} 0_{20} \\ 0 \\ 1 \end{pmatrix}, \quad \mu_{mag}^R = \frac{ma}{\sqrt{2}} \sinh \gamma \cosh \beta R_6 \begin{pmatrix} 0_4 \\ 0 \\ 1 \end{pmatrix}. \tag{2.18}$$

The superscripts  $L, R$  denote the left and right sectors respectively which are defined by  $A_a^{L(R)} = \frac{1}{2} (I_{28} - (+)L)_{ab} A_b$  for a vector  $A$ . The necessity of defining these vectors for left and right sectors separately is that for generic values of parameters  $\alpha, \beta$  and  $\gamma$ , the electric(magnetic) charge vectors are *not* parallel to each other and thus we will not be able to define the gyromagnetic (electric) ratios in general. We can apply various consistency

checks on our solution (2.2) - (2.18) . For example, if we set the parameter  $a$  to zero, we get the spherically symmetric configuration discussed in [1], on the other hand, setting  $\beta$  to zero, we get the purely electrically charged rotating black holes discussed in [3].

Since we have defined the charges and the dipole moments in the left and right sectors separately we are in a position to evaluate the gyromagnetic ( $g_{mag}$ ) and the gyroelectric ( $g_{elc}$ ) ratios for the left and right hand sectors independently. They are found to be

$$\begin{aligned} g_{mag}^L &= \frac{2\mu_{mag}^L M}{Q_{elc}^L J} = 2 \frac{\cosh\alpha \cosh\gamma + \cosh^2\beta}{(\cosh\alpha + \cosh\gamma)\cosh\gamma}, \\ g_{mag}^R &= \frac{2\mu_{mag}^R M}{Q_{elc}^R J} = 2 \frac{\cosh\alpha \cosh\gamma + \cosh^2\beta}{(\cosh\alpha + \cosh\gamma)\cosh\alpha} \end{aligned} \quad (2.19)$$

$$\begin{aligned} g_{elc}^L &= \frac{2\mu_{elc}^L M}{Q_{mag}^L J} = 2 \frac{(\cosh\alpha \cosh\gamma + \cosh^2\beta)\cosh\gamma}{(\cosh\alpha + \cosh\gamma)\cosh^2\beta}, \\ g_{elc}^R &= \frac{2\mu_{elc}^R M}{Q_{mag}^R J} = 2 \frac{(\cosh\alpha \cosh\gamma + \cosh^2\beta)\cosh\alpha}{(\cosh\alpha + \cosh\gamma)\cosh^2\beta} \end{aligned} \quad (2.20)$$

Here the superscript on  $g$  denotes the handedness as before. In the case of pure electrically charged black holes, as was noticed earlier [4,3], the gyromagnetic ratios have upper bound 2. Same is true for black holes in pure Einstein-Maxwell theory. On the other hand, in our case, the value of the gyromagnetic(electric) ratios *do not* seem to have any upper bound. Similar phenomenon was also noticed recently in [6] for the dyonic black holes in the Kaluza-Klein theory<sup>3</sup> To see it more explicitly let us study the following cases:

(1)  $\beta = \gamma$

$$g_{elc}^L = g_{mag}^L = 2, \quad g_{elc}^R = 2 \frac{\cosh\alpha}{\cosh\beta}, \quad g_{mag}^R = 2 \frac{\cosh\beta}{\cosh\alpha} \quad (2.21)$$

(2)  $\alpha = \gamma$

$$g_{elc}^{L,R} = 1 + \frac{\cosh^2\alpha}{\cosh^2\beta}, \quad g_{mag}^{L,R} = 1 + \frac{\cosh^2\beta}{\cosh^2\alpha} \quad (2.22)$$

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<sup>3</sup> However we differ from [6] in the following point. In our case by construction the electric and magnetic vectors are orthogonal. [6] on the other hand has only one  $U(1)$  with both electric and magnetic charge and hence parallel to each other. In a suitable  $O(6,22)$  frame, these charges in our case belong to different  $U(1)$ 's. Thus for dyonic black holes, our results are different from him by construction. However, if we restrict to either electrically charged or magnetically charged black holes, we can compare the results. For example, both reduces to Sen's result of purely electrically charged black holes. If, on the other hand, we restrict ourself only to the magnetically charged black hole ( $\alpha = \gamma = 0$  in our case), we find that the magnetically charged solution of [6] ( $\alpha = 0$  in his) and our's are the same after appropriate identifications of coordinates.

In the first case, we find that the left handed gyromagnetic (electric) ratios are equal to 2 but right handed gyromagnetic(electric) ratios can increase (decrease) as  $\beta$  ( $> \alpha$ ) increases and vice versa when  $\alpha$  ( $> \beta$ ) increases. In other words, the right handed gyromagnetic(electric) ratio effectively depends on the charge ratio  $\frac{|Q_{mag}^R|}{|Q_{elc}^R|}(\frac{|Q_{elc}^R|}{|Q_{mag}^R|})$ . Whenever one of the charge ratios gets large, gyromagnetic(or electric) ratio grows without bound. The chiralities will be exchanged if we replace  $\gamma$  by  $\alpha$ . In the second case however we find that the result is independent of chiralities but depends upon electric and magnetic nature of the ratios. In the limit of large  $\alpha$  or  $\beta$  either electric or magnetic ratio grows and the other one approaches 1. In other words, for large charges the gyromagnetic(electric) ratio behaves as  $1 + \frac{|Q_{mag}|}{|Q_{elc}|}(1 + \frac{|Q_{elc}|}{|Q_{mag}|})$ . So again, when one of the charge ratios becomes large, gyromagnetic (or electric) ratios grow without bound.

The singularity structure of the new solution remains the same as the original Kerr metric, i.e., the singularity appears at  $\rho = 0$ , and the inner and outer horizons are located at

$$\rho = m \pm \sqrt{m^2 - a^2} \quad (2.23)$$

In order to avoid naked singularity, we need to have  $a < m$ . The extremal limit is reached when  $a$  approaches  $m$ .

Now we will analyse various thermodynamic quantities. One such important quantity is the area of the event horizon as it determines the classical Bekenstein-Hawking entropy. We find

$$A = 4\pi m \cosh\beta (\cosh\alpha + \cosh\gamma)(m + \sqrt{m^2 - a^2}). \quad (2.24)$$

So the entropy is

$$S = \frac{A}{4} = \pi m \cosh\beta (\cosh\alpha + \cosh\gamma)(m + \sqrt{m^2 - a^2}). \quad (2.25)$$

The surface gravity which in our case is

$$\kappa = \frac{\sqrt{m^2 - a^2}}{m \cosh\beta (\cosh\alpha + \cosh\gamma)(m + \sqrt{m^2 - a^2})}. \quad (2.26)$$

Hence the temperature of the black hole is

$$T = \frac{\kappa}{2\pi} = \frac{\sqrt{m^2 - a^2}}{2\pi m \cosh\beta (\cosh\alpha + \cosh\gamma)(m + \sqrt{m^2 - a^2})}. \quad (2.27)$$

Comparing (2.14) and (2.25), we see that the entropy increases (decreases) with the increase (decrease) of angular momentum. On the other hand, the temperature decreases



with the increase of the angular momentum. However, the product of entropy and temperature is independent of the parameters  $\alpha$ ,  $\beta$  and  $\gamma$  and goes to zero in the extremal limit  $a \rightarrow m$ . Also the temperature vanishes in this extremal limit. This is quite similar to the case of purely electrically charged holes [3]<sup>4</sup>. A little analysis shows that for fixed  $m$  and  $a$ , the entropy and temperature can be expressed in terms of the electric and magnetic charges. However the expressions are not very illuminating so we do not display it here. But relations simplify as the charges become large and in this limit entropy always increases with the charges.

There are several limits of this class of solutions where it becomes supersymmetric and BPS saturated. But in all these cases, one has to set the rotation parameter  $a$  to zero otherwise the horizons disappear leaving behind a naked singularity. This fact has been observed earlier also[8]. Therefore the supersymmetric solution reduces to static spherically symmetric dyonic black hole. Analysis of these black holes have already been presented in our earlier paper [1]. We expect that our results will be useful to get a better understanding of the relationship between black holes and elementary string states in the same spirit of refs.[9].

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<sup>4</sup> However, for the dyonic black holes with unconstrained left, right magnetic charges, entropy is *finite* for spherically symmetric case as can be seen in [7,1]

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